
# Applications of copula-based model for multivariate analysis of compound extreme events and development of financial risk transfer

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# Compound extremes

- The simultaneous impacts of compound events (CEs), compared to single ones, have the potential to cause higher socioeconomic and environmental losses.
- Australia has many different climate zones and is known to experience many types of hazards in short timeframes, including heatwaves, (flash) droughts, bushfires, or floods.
- Australia has been identified as exceptionally susceptible to the occurrence of compound drought and heatwaves[1].
- For example: compound hot and dry summers occur particularly frequently in large parts of northern Australia[2]. In the past 150 years, an increasing frequency of dry and hot months has been reported in Southeast Australia. Recent works have analysed compound heatwave and drought hotspots and their trends in the southeast region[3].

# Compound extremes

- Very few extreme event combinations, such as drought and heatwave, strong winds, and heavy precipitation, are covered in Australia[5,6].
- Yet, many other aspects such as drought and fire risk, heatwave and ozone, warm and humid events, and drought and aridity, for which several parts of Australia have been deemed as global hotspots, have not yet been addressed.

**Table 1. List of event combinations of drought and heatwave-associated compound events (DHCEs) has been published[4].**

| (DHCEs) | Global | Australia |
|---|---|---|
| Meteorological-agricultural-hydrological drought | | |
| Flash drought-heatwave | | |
| Drought-heatwave-fire risk | 2 | |
| Heatwave-ozone | 1 | |
| Day-night time heatwave | | |
| Dry-cold | 1 | |
| Warm-humid | 3 | |
| Wet-dry | 2 | |
| Warm-wet | 3 | |
| Precipitation-temperature | 6 | |
| Drought-aridity | 2 | |
| Drought-heatwave | 25 | 3 |

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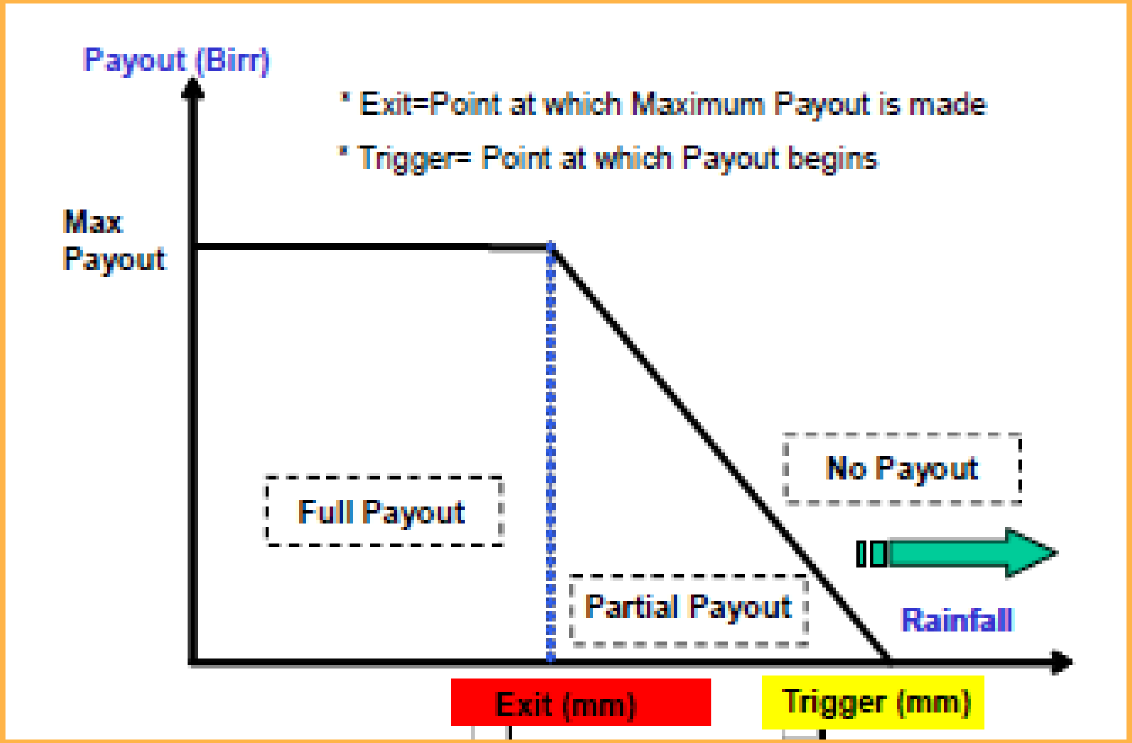
# Catastrophic risk reduction through index insurance

- The insurance pays out to the policy holder on the basis of a predetermined index (e.g. rainfall or drought level) for losses (e.g. crop yield) resulting from weather and catastrophic events (e.g. floods or droughts).
- Advantages: no need claims assessors, quicker and more objective claim settlement processes, lower adverse selection, moral hazard, systemic risks and administrative costs.



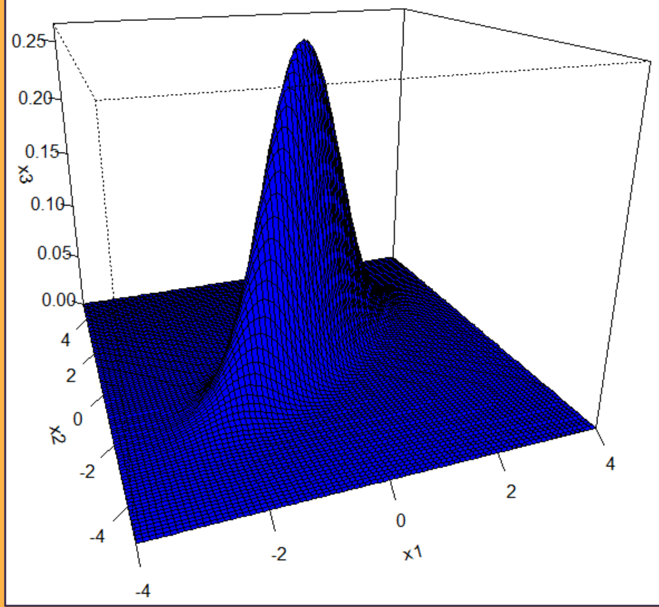

Source: RERR Final Report, January 2016, JICA

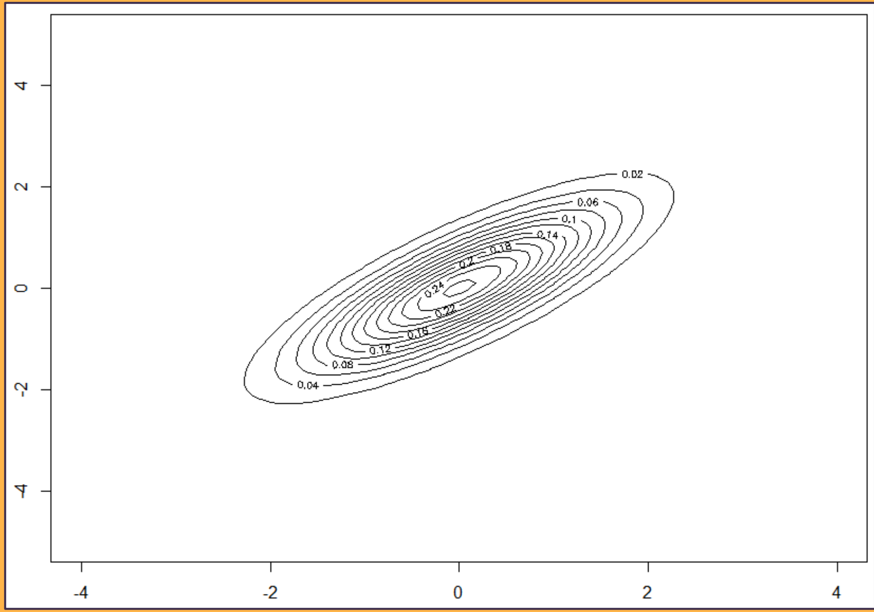
# Catastrophic risk reduction through index insurance

- Challenges: basis risks (design, temporal, and geographical), refers to the imperfect correlation between index insurance and the loss variables, leading to high premiums.
- Index insurance contracts designed based on classic piecewise-linear models inadequately reflect the yield loss.
- Non-linear statistical methods generalized additive regressions, quantile regressions and **copulas** can address extremes in index insurance.


# Copulas

- The copula approach to multivariate data allows to model the margins individually.
- The shape of scatter plots between two variables depends on the scale of the variables.
- We'd like to standardise each variable to see if there's dependence between standardised variables. The **probability integral transform (PIT)** can be used.
- This approach separates the dependence between the components from the marginal distributions.
- For this, the dependence between random variables with uniform margins has to be modelled by a corresponding joint distribution function which is called **a copula**.



Densities (a) and contour lines (b) of bivariate standard normal distribution with correlation of .8.

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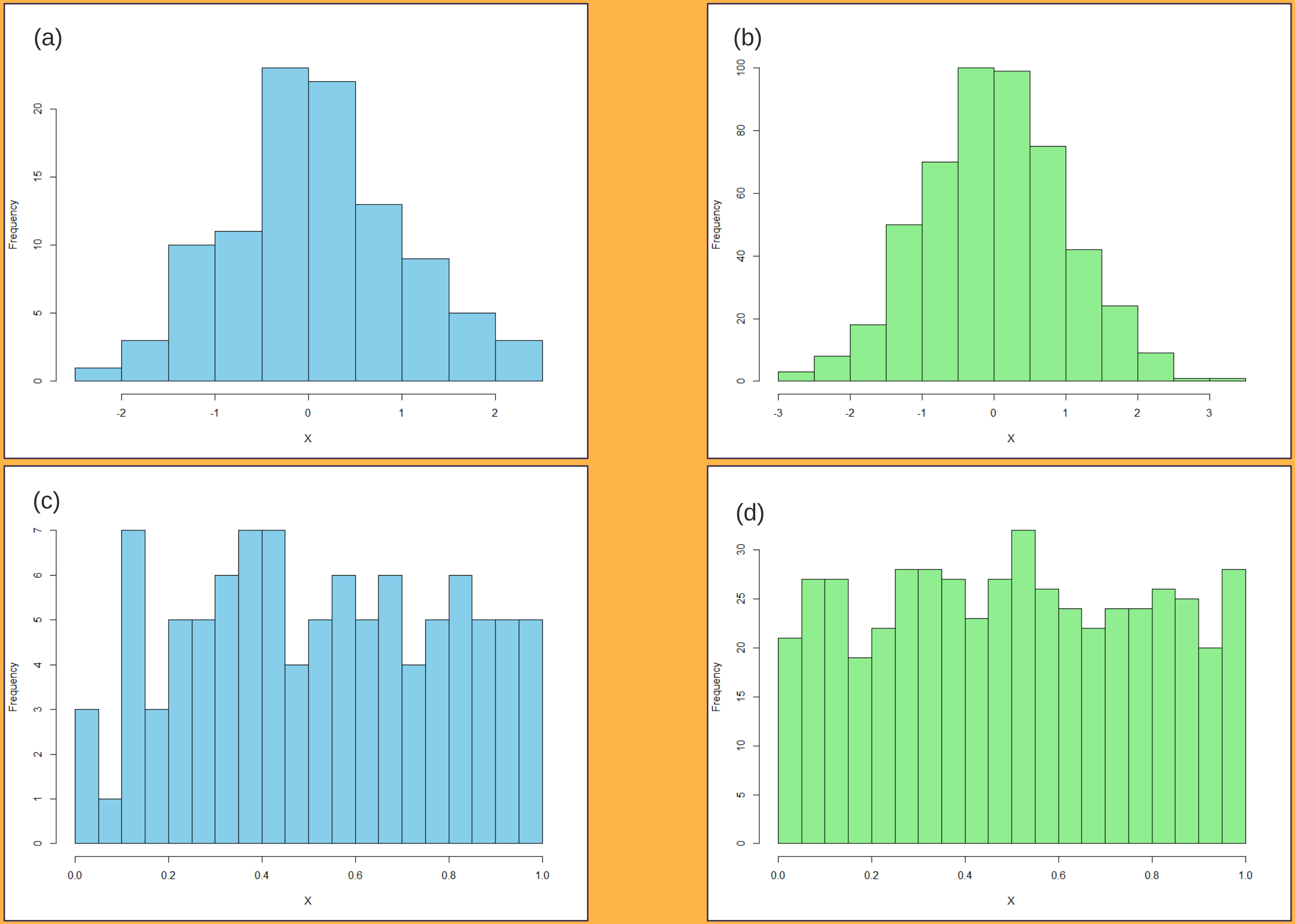
# Probability integral transform (PIT)

- If $X \sim F$ is a continuous random variable and $x$ is an observed value of $X$, then the transformation $u = F(x)$ is called the PIT at $x$.
- Distribution of the PIT: If $X \sim F$ then $U = F(X)$ is uniformly distributed, as:

$$P(U \leq u) = P(F(X) \leq u) = P(X \leq F^{-1}(u)) = F(F^{-1}(u)) = u$$

  holds for every $u \in [0,1]$.
- If F is estimated by parametric distributions or by the empirical distribution, then this holds only approximately.




Histograms of a standard normal random sample of size 100 (a) and 500 (b) and their associated PIT values using the standard normal distribution function (c-d).

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# Copulas

- Sklar's theorem: Let X be a d-dimensional random vector with joint distribution function F and marginal distribution functions $F_i$, i = 1,...,d, then the joint distribution function can be expressed as

$$F(x_1, ..., xd) = C(F_1(x_1), ..., Fd(xd))$$

with associated density or probability mass function

$$f(x_1, ..., x_d) = c(F_1(x_1), ..., F_d(x_d))f_1(x_1)...f_d(x_d)$$

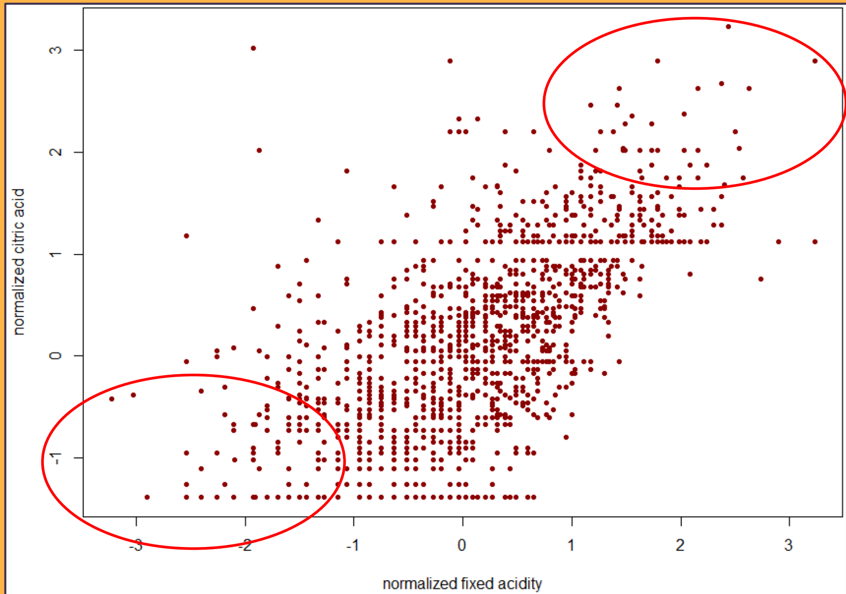
for some d-dimensional copula C with copula density c. For absolutely continuous distributions, the copula C is unique.

- The inverse also holds: the copula corresponding to a multivariate distribution function F with marginal distribution functions $F_i$ for i =1,...,d can be expressed as

$$C(u_1, ..., u_d) = F(F_1^{-1}(u_1), ..., F_d^{-1}(u_d))$$

and its copula density or probability mass function is determined by

$$c(u_1, \ldots u_d) = \frac{f(F_1^{-1}(u_1), ..., F_d^{-1}(u_d))}{f_1(F_1^{-1}(u_1)), \ldots, f_d(F_d^{-1}(u_d))}$$

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# Bivariate Copulas

The simulated joint cumulative distribution function (JCDF) and the probability density function (JPDF) of bivariate Elliptical (Gaussian) copulas with differently parameters[7], θ and τ.

JCDF plots (a-c)

(a) Gaussian: θ=0.23; τ=0.15

(b) Gaussian: θ=0.45; τ=0.30

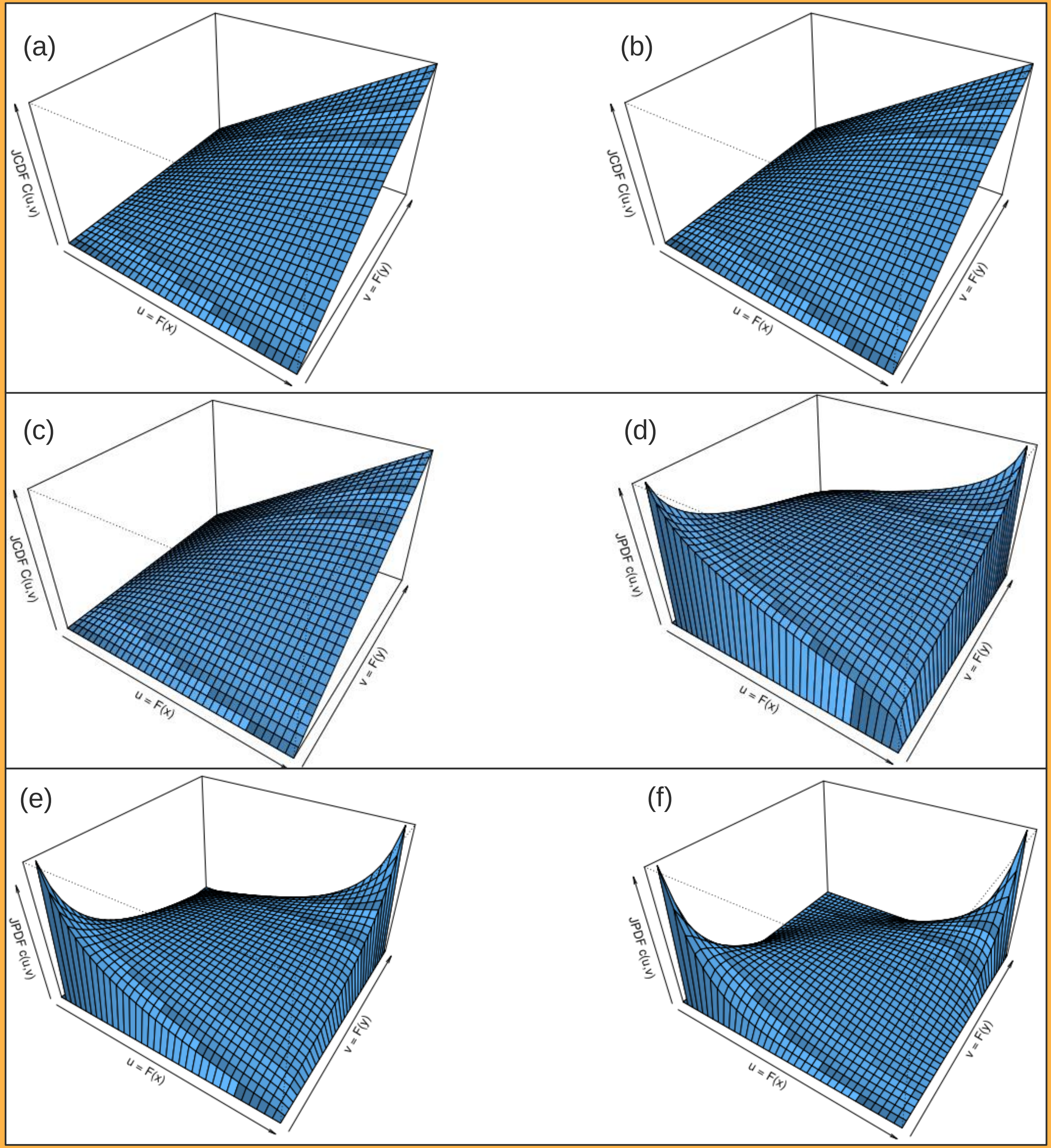
(c) Gaussian: θ=0.71; τ=0.60

JPDF plots (d-f)

(d) Gaussian: θ=0.23; τ=0.15

(e) Gaussian: θ=0.45; τ=0.30

(f) Gaussian: θ=0.71; τ=0.60

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# Bivariate Copulas

The simulated joint cumulative distribution function (JCDF) and the probability density function (JPDF) of bivariate Archimedean copulas models with differently parameters, θ and τ .

JCDF plots [a-d]

[a] Clayton: θ=2.00; τ=0.50

[b] Gumbel: θ=2.00; τ=0.50

[c] Frank: θ=2.00; τ=0.50

[d] Joe: θ=2.86; τ=0.50

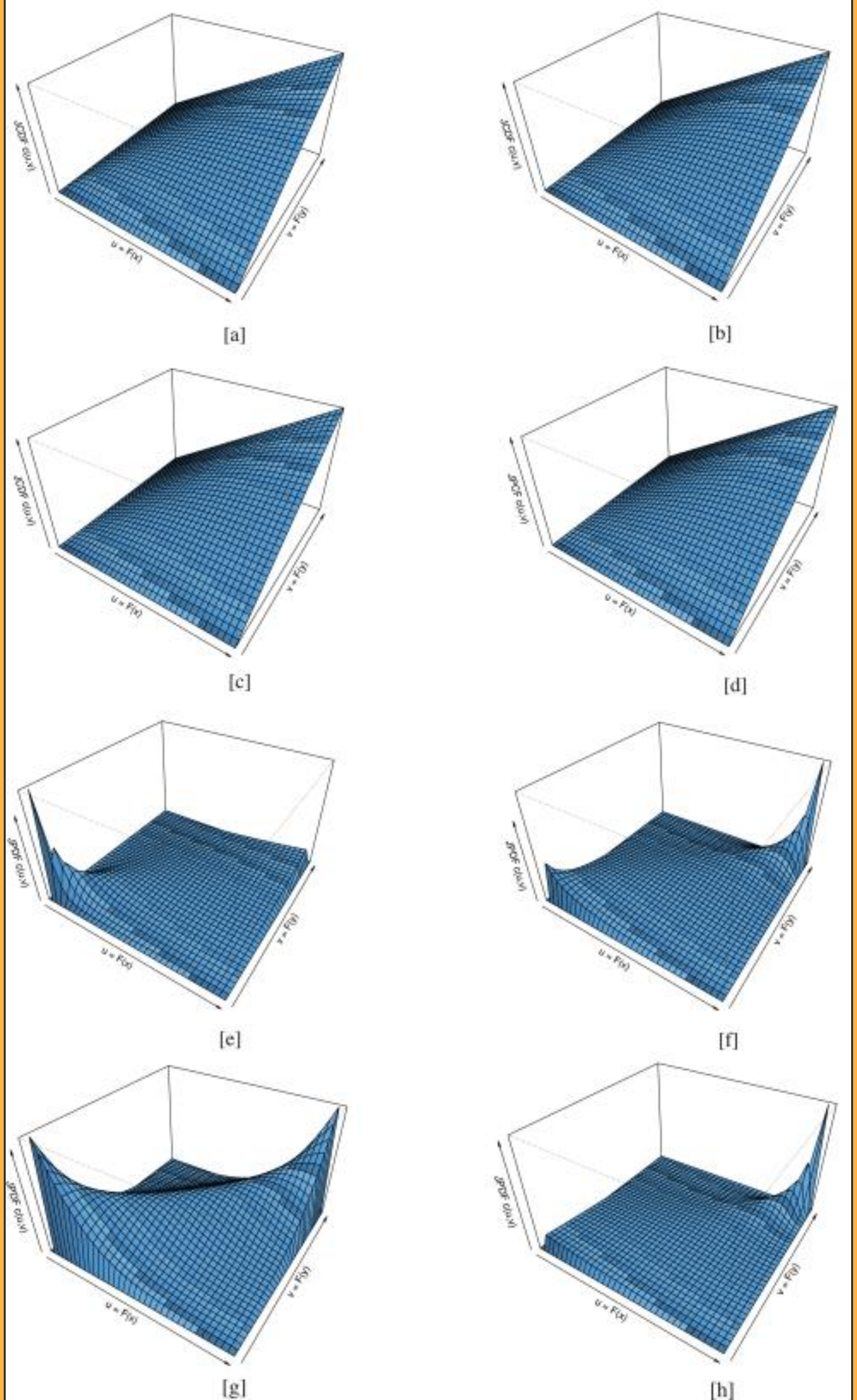
JPDF plots [e-h]

[e] Clayton: θ=2.00; τ=0.50

[f] Gumbel: θ=2.00; τ=0.50

[g] Frank: θ=2.00; τ=0.50

[h] Joe: θ=2.86; τ=0.50

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# Copula-based modelling of meteorological and agricultural droughts

- The 3-month standardised precipitation evapotranspiration index (SPEI-3) was calculated using monthly gridded precipitation data (5km) downloaded from SILO - DES. The SPEI-3 was derived by fitting observed precipitation to a gamma distribution.

| **Value Range** | SPEI ≤ -2 | -2 < SPEI ≤ -1.5 | -1.5 < SPEI ≤ -1 | -1 < SPEI ≤ -0.5 | SPEI > -0.5 |
|---|---|---|---|---|---|
| **Levels** | Exceptional drought | Severe drought | Middle drought | Moderate drought | Non-drought |

- The monthly gridded soil moisture (SM) dataset (5km) was downloaded from BoM's AWRA-L, version 6.0 model. The soil moisture dataset is "root zone soil moisture", which represents the percentage of available water content in the top 1 m of the soil profile. The SM data was converted to percentiles using maximum and minimum values.

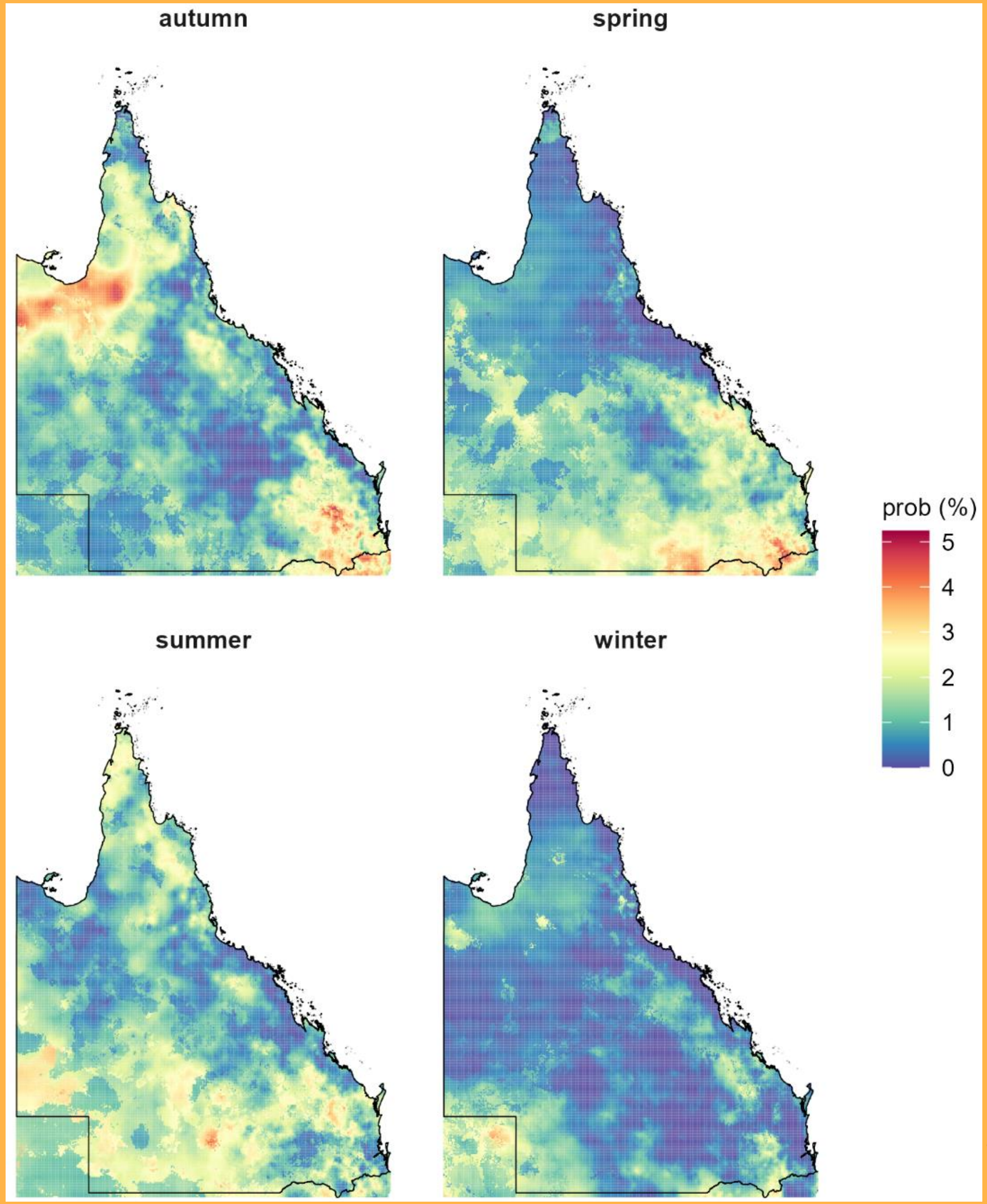
| Category | Description | Soil moisture percentiles |
|---|---|---|
| None | Typical to wet | >31 |
| D0 | Abnormally dry | 21-31 |
| D1 | Moderate drought | 11-20 |
| D2 | Severe drought | 6-10 |
| D3 | Extreme drought | 3-5 |
| D4 | Exceptional drought | 0-2 |

# Copula-based modelling of meteorological and agricultural droughts

- The figure describes the joint probability that meteorological droughts and agricultural droughts occur simultaneously, i.e. P(SPEI-3 < 1.5 & SM < 0.1), in the four seasons in QLD.
- Can be used to derive the joint return period:

$$T = \frac{1}{1 - P(X \le x, Y \le y)} = \frac{1}{1 - C\left(F_x(x), F_y(y)\right)}$$

- The joint probability and return period can be used in the design of index insurance.



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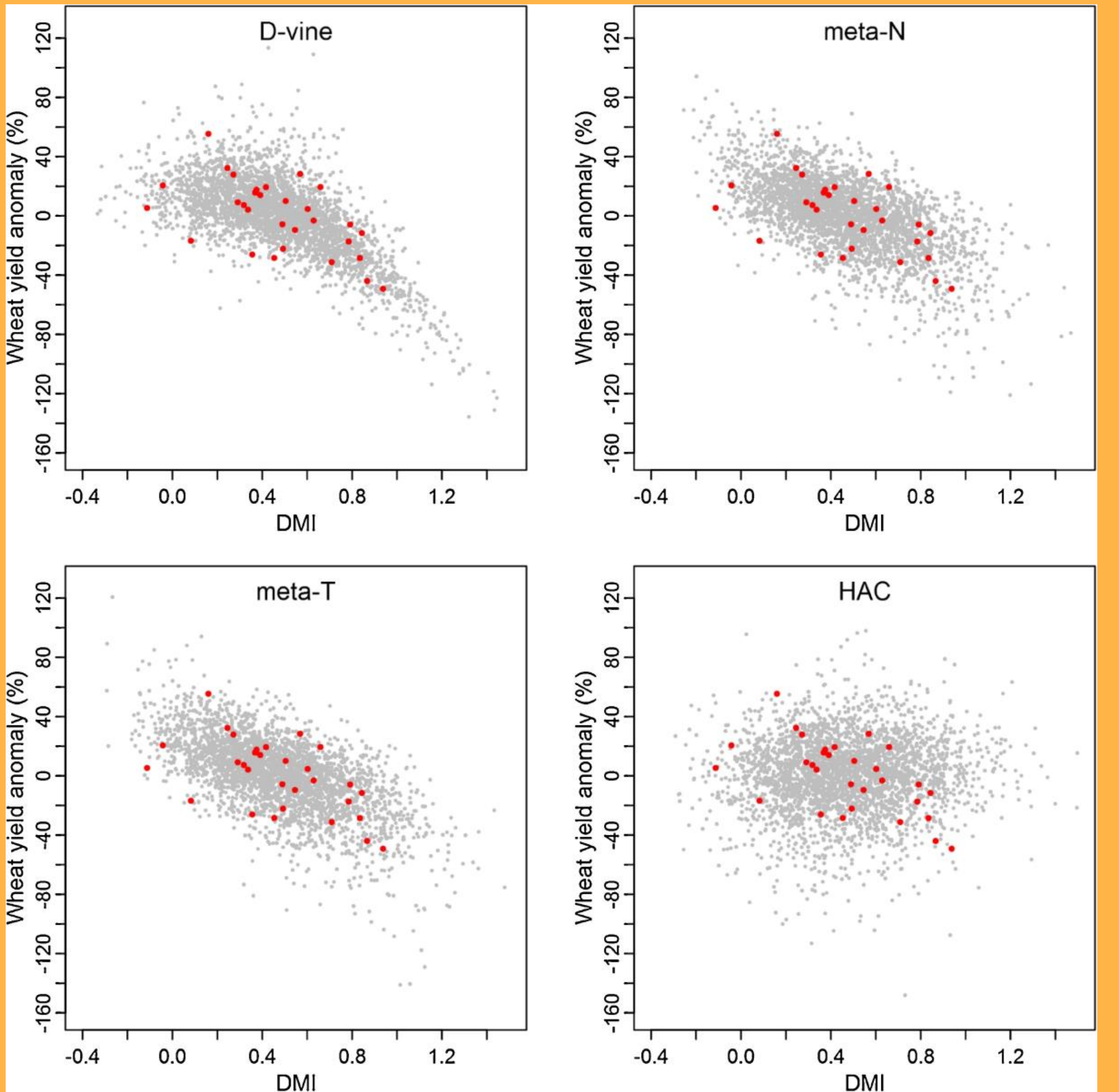
# Catastrophic risk reduction through index insurance derived by the copula approach

- The copula approach is better suited for modelling tail dependence than the standard linear correlation approach.

Comparisons between the observed and simulated averaged March–May DMI and QLD percentage wheat yield anomalies for different three-dimensional copulas: vine (D-vine), meta-Gaussian (meta-N), meta-Student T (meta-T), and hierarchical Archimedean copula (HAC)[8].

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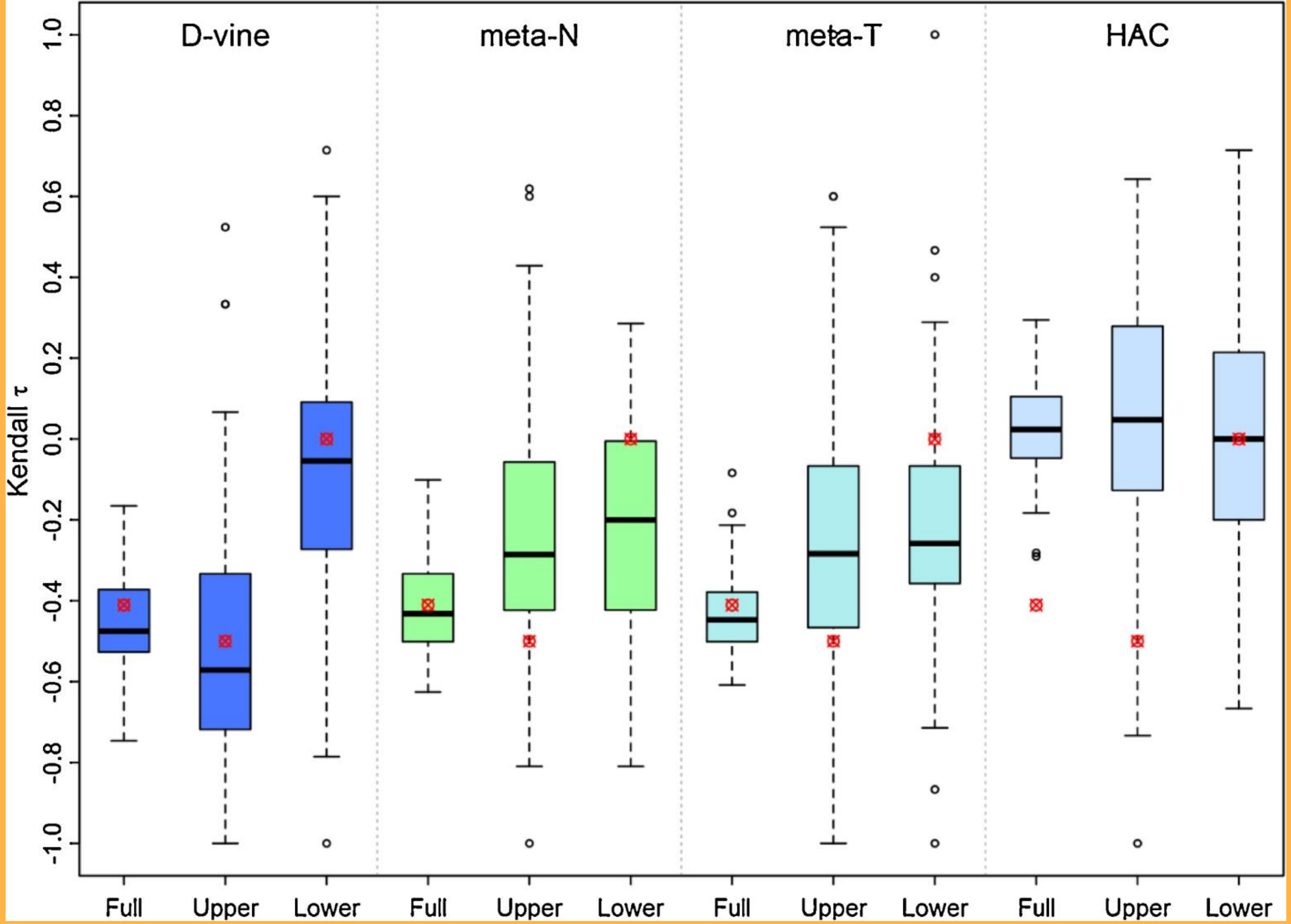


Comparisons of the observed and simulated Kendall’s tau between averaged March-April-May DMI and QLD percentage wheat yield anomalies from different three-dimensional copulas as in Fig. 4. Cases are considered for the whole data (Full), the upper tail (Upper) and the lower tail (Lower)[8].

# Catastrophic risk reduction through index insurance derived by the copula approach

- May increase the effectiveness of weather insurance contracts designed to provide protection against extreme weather events. For example:
  - ✓ To design and rate a weather index insurance contract based on the concept of the marginal expected shortfall (MES), defined as the crop yield $\tilde{\mu}$ conditioned on the realisation of the weather index **W** below some selected quantiles $q_\alpha$ of its distribution:

$$\tilde{\mu} = MES_\alpha Y \middle| W = E\left[ Y \middle| W \leq q_\alpha (W) \right]$$

  - ✓ To determine $\tilde{\mu}$, we must define the conditional distribution of the yield variable given weather variable single realisations (**W=w**) and express it in terms of a **conditional copula**:

$$H_{Y|W=w}(y) = c_{F(Y)|F(W=w)}(v) \Big|_{F(W)=u}^{F(Y)=v}$$

where **F(W)** and **F(Y)** are the marginal distributions of the weather index and crop yield, respectively, and **u** and **v** are the corresponding PIT.

  - ✓ The **indemnity ($I_t$)** is defined conditional on weather index realisation as the difference between the strike or reference yield **y** and $\tilde{\mu}$ for the year **t**: $I_t = \left[ y - \tilde{\mu} \middle| W_t \leq q_\alpha (W), 0 \middle| W_t > q_\alpha (W) \right]$

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